

Exercise 19

The quantity of charge Q in coulombs (C) that has passed through a point in a wire up to time t (measured in seconds) is given by $Q(t) = t^3 - 2t^2 + 6t + 2$. Find the current when (a) $t = 0.5$ s and (b) $t = 1$ s. [See Example 3. The unit of current is an ampere ($1 \text{ A} = 1 \text{ C/s}$).] At what time is the current lowest?

Solution

Take the derivative of $Q(t)$ to get the current in the wire.

$$\begin{aligned}\frac{dQ}{dt} &= \frac{d}{dt}(t^3 - 2t^2 + 6t + 2) \\ &= 3t^2 - 4t + 6\end{aligned}$$

The current when $t = 0.5$ s is

$$\left. \frac{dQ}{dt} \right|_{t=0.5} = 3(0.5)^2 - 4(0.5) + 6 = 4.75 \text{ C/s.}$$

The current when $t = 1$ s is

$$\left. \frac{dQ}{dt} \right|_{t=1} = 3(1)^2 - 4(1) + 6 = 5 \text{ C/s.}$$

To find when the current is lowest, take the derivative of the current,

$$\frac{d^2Q}{dt^2} = \frac{d}{dt}(3t^2 - 4t + 6) = 6t - 4,$$

set it equal to zero,

$$6t - 4 = 0,$$

and solve for t .

$$t = \frac{2}{3}$$